Mid-Term Exam Algebra (12– 4 –2014) Prep-Year Model Answer

[1](a)A square matrix A is called symmetric if $\mathbf{A^t = A}$.

(b)A square matrix A is called singular if $|\mathbf{A}| = \mathbf{0}$.

(c)A square matrix A has inverse A^{-1} if $|\mathbf{A}| \neq \mathbf{0}$.

-- ***(3 Marks)***

[2] If $A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \end{bmatrix}$

(a) A + B does not exist, B.A does not exist. $A.B = \begin{bmatrix} 10 & 6 & 0 \\ -1 & 3 & 3 \end{bmatrix}$

-- ***(3 Marks)***

(b) $|A - \lambda I| = \begin{vmatrix} 2-\lambda & 4 \\ 1 & -1-\lambda \end{vmatrix} = (2-\lambda)(-1-\lambda) - 4 = 0$

Then, $\lambda^2 - \lambda - 6 = 0$. Then, the eigenvalues: $\lambda_1 = 3$, $\lambda_2 = -2$.

From the equation, $\begin{bmatrix} 2-\lambda & 4 \\ 1 & -1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = 3$, $\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then – x + 4y = 0, x – 4y = 0

Then x = 4y = any number except 0

Put y = 1, we get x = 4 and the

corresponding eigenvector is: $X_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

For : $\lambda_2 = -2$, $\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then 4x + 4y = 0, x + y = 0

Then y = – x = any number except 0

Put x = 1, we get y = – 1 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

-- ***(5 Marks)***

(c) The eigenvalues of $f(A) = A^3 - 2A$ are $f(3) = 27 - 6 = 21$, $f(-2) = -8 + 4 = -4$.

-- ***(2 Marks)***

(d) The eigenvalues of $f(A) = 2^A$ are $f(3) = 2^3 = 8$, $f(-2) = 2^{-2} = \frac{1}{4}$.

-- ***(2 Marks)***

(e)Since $T = \begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix}$. Then $T^{-1} = -\frac{1}{5}\begin{bmatrix} -1 & -1 \\ -1 & 4 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix}$

Since $T^{-1}.A.T = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$

Then $A^n = T.\begin{bmatrix} 3^n & 0 \\ 0 & (-2)^n \end{bmatrix}.T^{-1} = \begin{bmatrix} 4 & 1 \\ 1 & -1 \end{bmatrix}.\begin{bmatrix} 3^n & 0 \\ 0 & (-2)^n \end{bmatrix}.\frac{1}{5}\begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix}$

$$= \frac{1}{5}\begin{bmatrix} 4(3)^n + (-2)^n & 4(3)^n - 4(-2)^n \\ (3)^n - (-2)^n & (3)^n + 4(-2)^n \end{bmatrix}$$

-- ***(5 Marks)***

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